Walking motion generation with online foot position adaptation based on ℓ_1- and ℓ_∞-norm penalty formulations

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Scenario

- humanoid robot walks on a $\underbrace{\text{flat surface}}_{\text{assumption}}$
- the system is subject to external disturbances
- higher level planner generates reference footsteps

Objective

follow reference footsteps (exactly, when possible) while preserving the "stability" of the system

Required

a scheme for online trajectory following and stabilization

Trajectory following + stabilization $\triangleq$ walking motion generation

How to approach the problem (in under 10 ms)

- using predefining motion primitives - not possible in the presence of disturbances
- making local decisions considering the full dynamical morel - not reliable
- "look-ahead" schemes - increasingly popular but computationally demanding. In particular, using full system dynamics - not feasible

One possible "solution"

- use approximate dynamical model (preferably linear)
- compensate the approximation by applying a preview type of controller with (possibly) fast sampling rate

We use ...

- **model**: linearized 3D inverted pendulum - surprisingly accurate approximation (under certain assumptions)
- **preview controller**: Linear Quadratic Regulator (LQR) with explicit constraints $\triangleq$ Linear Model Predictive Control (LMPC)
- **stability criterion**: ZMP $\in$ support polygon

Explicit constraints - address the stabilization sub-task

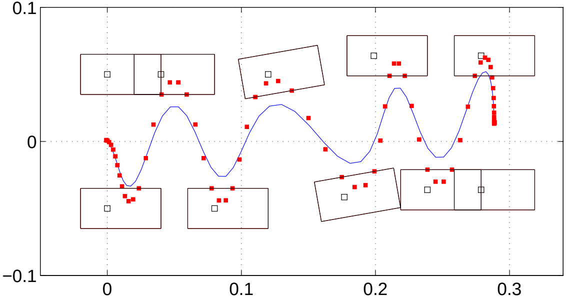
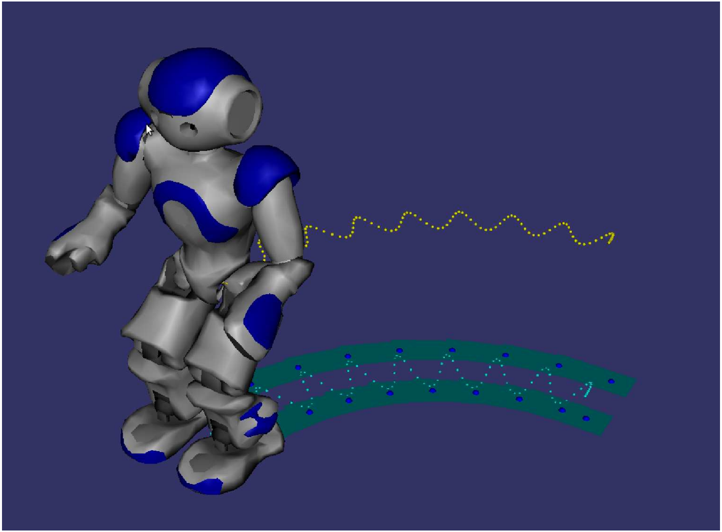


Figure: A typical result (fixed feet). Red squares - ZMP, blue line - CoM



Given CoM & feet positions **solve** inverse kinematics to apply necessary control input ...

The paper deals with ...

Main contribution

- walking motion generation
  generate "safe" motion profile for the CoM
- online foot position adaptation
  compute "optimal" foot repositioning "when necessary" (due to disturbances)
- ℓ_1- and ℓ_∞-norm penalty
  define "optimal" and "when necessary" (and motivate them)

In addition ...

- change of variable that leads to a simplified formulation
- double support handling with foot adaptation

Typical result - foot variation (no disturbance)

Relax footstep constraints: penalize quadratic ℓ_2-norm of foot variation

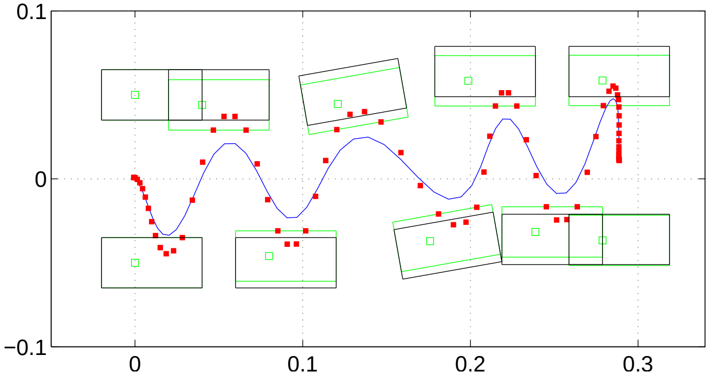


Figure: Reference footsteps altered even though it is "not necessary"

Fact

Regardless of how large (finite) quadratic ℓ_2 penalty is used, footstep repositioning would occur

What penalty to use?

We are not the first people to ask this question :)

A wide variety of options are available. Popular ones:

- ℓ_1-norm based penalization
- ℓ_∞-norm based penalization

reason: discontinuity at the origin leads to the following property

Minimization of ℓ_1-norm or ℓ_∞-norm tends to produce sparse solutions.

Heavily used in

- compressed sensing
- approximate solution of cardinality problems
- robust (to outliers, or noise) estimation in statistics
- sparse signal reconstructions
- optimization methods using exact penalization
- imposing **soft constraints** in the context of MPC
- classification in machine learning ($e.g.$, soft margin SVM), etc.

Sparse solutions

Suppose that $\Delta \boldsymbol{F}_i$ represents foot variation from the reference position for the i^{th} footstep in the preview window, then

$$\begin{aligned} \text{minimize} \ \ & \textbf{usual stuff} \ + \ \alpha(|\Delta \boldsymbol{F}_1| + \cdots + |\Delta \boldsymbol{F}_k|) \\ \text{subject to} \ \ & \textbf{usual stuff} \end{aligned}$$

leads to foot repositioning only "when necessary". Define what "when necessary" means by the gain $\alpha > 0$.

Relation: $\alpha \leftrightarrow$ set of disturbances that do not lead to foot repositioning.

Many interesting options to consider

The paper presents two formulations (require solving a single QP)

- quadratic ℓ_2-norm + ℓ_1-norm penalty $\rightarrow$ (slightly more variables)
- quadratic ℓ_2-norm + ℓ_∞-norm penalty $\rightarrow$ (slightly more constraints)

"Shaping-up" alternative norms is possible with both formulations ...

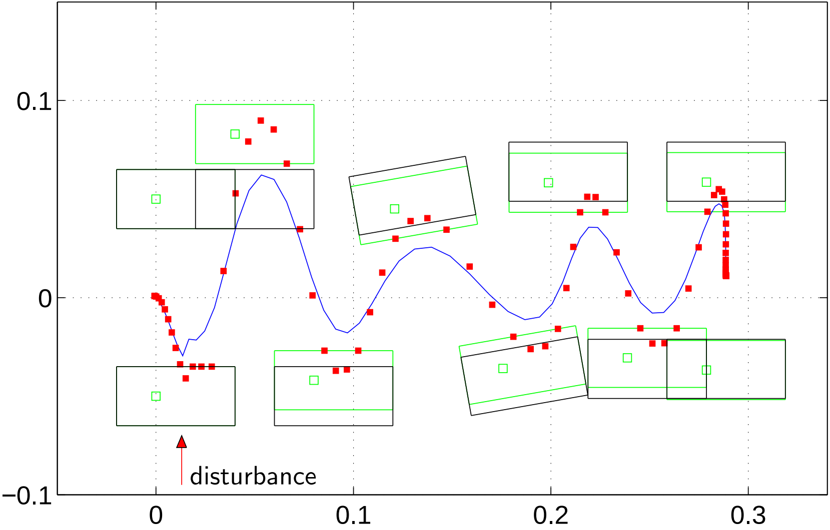


Figure: ℓ_2-norm minimization

Example (with disturbance)

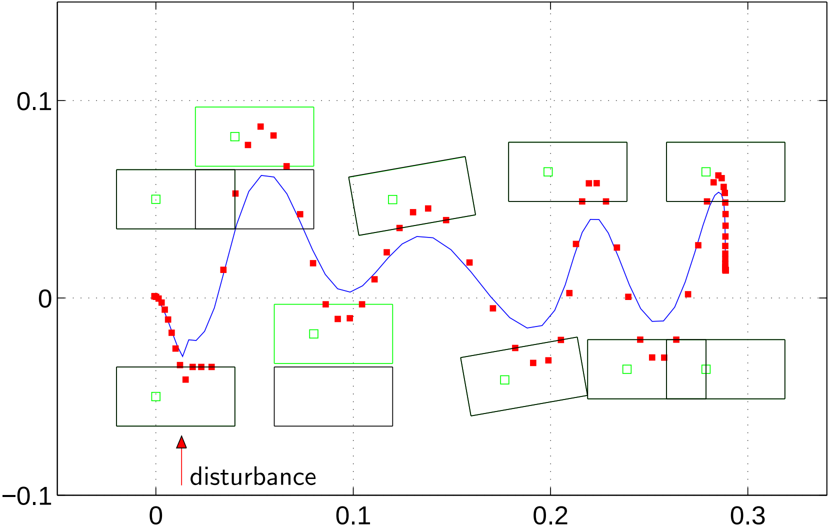


Figure: ℓ_2-norm in combination with ℓ_∞-norm minimization.

Change of variable that leads to a simplified formulation

Standard approach

- **control input**: jerk of CoM
- **output**: position of ZMP

position of ZMP $\leftarrow$ system dynamics $\leftarrow$ jerk of CoM

$\Rightarrow$ The system dynamics appears in the constraints for the ZMP

We use the ZMP directly as a decision variable

$$\begin{aligned} \underset{\text{ZMP}}{\text{minimize}} \quad & \textbf{usual stuff} \\ \text{subject to} \quad & \text{ZMP} \in \text{support polygon} \leftarrow \text{pure geometry} \end{aligned}$$

In this way we can derive a formulation with simple bounds

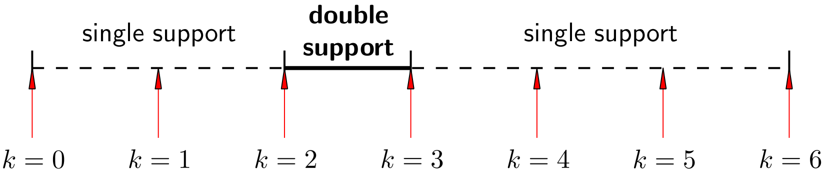
Double support handling with foot adaptation

Foot variation allowed $\Rightarrow$

relation between $\Delta \boldsymbol{F}_i$ and the double support constraint is nonlinear.

To circumvent this problem

LMPC schemes that perform foot adaptation assume that no sampling times fall strictly in double support (*i.e.*, they jump-over the double support phase) $\rightarrow$ does not scale well with some walking patters ...



In the context of foot adaptation, we present a "reasonable" approximate way to account for double support constraints

Future work ...

Two ways of implementing this LMPC scheme

- sequential formulation: usually dense, with less variables. Objective function can be formed **offline**. The use of off-the-shelf dense solvers possible (more appealing to practitioners)

this paper

- simultaneous formulation: usually sparse, with more variables. **No need to explicitly form an objective function**. The use of specialized solvers necessary ... (many possibilities)
  We allow
  - **variable sampling time**
  - **variable CoM height**, etc.

next IROS

thank you